

chain-consistent-prior-to-input^{13,45}

$\forall es:ES, Cmd:Type, In:AbsInterface(Cmd), isupdate:(Cmd \rightarrow \mathbb{B}), Sys, Out:AbsInterface(Top).$
 $(E(In) \subseteq_r E(Sys))$

$\Rightarrow (\forall f:sys\text{-antecedent}(es;Sys), chain:(E(Sys) \rightarrow (Id\ List)).$

$chain\text{-consistent}(f;chain)$

$\Rightarrow (\forall a:E(In), b:E(Sys).$

$(loc(b) = loc(a) \in Id)$

$\Rightarrow (\neg(\uparrow(b \in_b In)))$

$\Rightarrow (\uparrow(isupdate(In(a))))$

$\Rightarrow (\neg(loc(f(b)) = loc(b) \in Id))$

$\Rightarrow b \leq_{loc\ prior}(Sys)(a))$